

Answers to Homework 1

Multivariable Calculus

1 Derivatives

Find the derivative of the following functions:

1. $f(x) = 37$ Ans: $f'(x) = 0$.

2. $f(x) = 5x^2 + 3x - 2$ Ans: $f'(x) = 10x + 3$.

3. $f(x) = \frac{1}{x^2}$ Ans: $f'(x) = \frac{-2}{x^3}$.

4. $f(x) = x + \sqrt{x}$ Ans: $f'(x) = 1 + \frac{1}{2x^{\frac{1}{2}}}$.

5. $f(x) = \sqrt{x}(x - 1)$ Ans: $f'(x) = \frac{3}{2}\sqrt{x} - \frac{1}{2x^{\frac{1}{2}}}$.

6. $f(x) = \sqrt{1 + 2x}$ Ans: $f'(x) = \frac{1}{\sqrt{1 + 2x}}$.

7. $f(x) = \frac{1 - 3x}{1 + x}$ Ans: $f'(x) = \frac{-4}{(1 + x)^2}$.

8. $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$ Ans: $f'(x) = \frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} - \frac{3}{2x^{\frac{3}{2}}}$.

9. $f(x) = e^{x+1} + 1$ Ans: $f'(x) = e^{x+1}$.

Product rule

10. $f(x) = x^2e^x$ Ans: $f'(x) = x^2e^x + 2xe^x$.

11. $f(x) = \frac{e^x}{1 + x}$ Ans: $f'(x) = \frac{xe^x}{(1 + x)^2}$.

12. $f(x) = (x + e^x)(\sin x)$ Ans: $f'(x) = (1 + e^x)\sin x + (x + e^x)\cos x$.

Trigonometric function

13. $f(x) = x \sin x$ Ans: $f'(x) = \sin x + x \cos x$.

14. $f(x) = e^x(\cos x + x)$ Ans: $f'(x) = e^x(\cos x + x) + e^x(-\sin x + 1)$.

15. $f(x) = \frac{\sec x}{1 + \sec x}$ Ans: $f'(x) = \frac{\sin x}{(\cos x + 1)^2}$.

16. $f(x) = x(\sin x)(\cos x)$ Ans: $f'(x) = \sin x \cos x + x(\cos x)^2 - x(\sin x)^2$.

Chain rule

17. $f(x) = \tan(\sin x)$ Ans: $f'(x) = (\sec^2(\sin x)) \cdot \cos x$.

18. $f(x) = (1 - x^2)^{10}$ Ans: $f'(x) = 10(1 - x^2)^9 \cdot (-2x)$.

19. $f(x) = \sin(e^x)$ Ans: $f'(x) = e^x \cos(e^x)$.

20. $f(x) = e^{x \cos x}$ Ans: $f'(x) = (\cos x - x \sin x)e^{x \cos x}$.

21. $f(x) = x \sin(\frac{1}{x})$ Ans: $f'(x) = \sin(\frac{1}{x}) - \frac{1}{x} \cos(\frac{1}{x})$.

2 Integrals

1. $\int (1 + 3x) dx = x + \frac{3x^2}{2} + C$.

2. $\int (1 - 2x^3) dx = x - \frac{x^4}{2} + C$.

3. $\int x^{-\frac{3}{4}} dx = 4x^{\frac{1}{4}} + C$.

4. $\int (2 - \sqrt{x})^2 dx = 4x + \frac{x^2}{2} - \frac{8x^{\frac{3}{2}}}{3} + C$.

5. $\int_{-1}^0 (2x - e^x) dx = -2 + e^{-1}$.

U-substitution

6. $\int (3x - 2)^{200} dx = \frac{(3x - 2)^{201}}{603} + C$.

7. $\int (\cos 3x) dx = \frac{\sin(3x)}{3} + C$.

8. $\int \frac{x}{(x^2 + 1)^2} dx = -\frac{1}{2(x^2 + 1)} + C$.

9. $\int \sqrt{4 - x} dx = -\frac{2}{3}(4 - x)^{\frac{3}{2}} + C$.

10. $\int \frac{dx}{5 - 3x} = -\frac{1}{3} \ln |5 - 3x| + C$.

11. $\int \frac{dx}{x \ln x} = \ln |\ln(|x|)| + C$.

12. $\int e^{\cos x} \sin x dx = -e^{\cos x} + C$.

13. $\int \sin(\pi x) dx = -\frac{\cos(\pi x)}{\pi} + C$.

14. $\int \frac{x}{\sqrt{x^2 + 1}} dx = \sqrt{x^2 + 1} + C$.

$$15. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2e^{\sqrt{x}} + C.$$

$$16. \int \sin x \cdot \cos(\cos x) dx = -\sin(\cos(x)) + C.$$

Integration by Parts

$$17. \int x \sin x dx = \sin x - x \cos x + C.$$

$$18. \int x^2 e^x dx = (x^2 - 2x + 2)e^x + C.$$

$$19. \int x \ln x dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C.$$

Trigonometric Integrals

$$20. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C.$$

$$21. \int \sin^3 x dx = \frac{\cos^3 x}{3} - \cos x + C.$$

Trigonometric Substitution

$$22. \int x \sqrt{1-x^2} dx = -\frac{(1-x^2)^{\frac{3}{2}}}{3} + C.$$