First Name_____ Last Name_____ ID. _____

Mahidol University International College

Final Examination

Trimester 2/ 2019-2020

Course/Code: General Mathematics II/ICMA 212

Instructor : Aj. Thotsaporn THANATIPANONDA

Date:	23 March 2020
Time:	10:00 - 11:50pm
Total pages:	Exam 7 pages

Directions

1. There are 6 questions. Total score is 55 points which scale to 35% of total grade.
2. Students are allowed to use/consult a calculator, notes and old homeworks.
3. Show a reasonable amount of work. **Simplify as much as you can.** There will be no credit for an answer alone.
4. Students found consulting with other people or internet during the examination will be penalized according to the university's examination policy.

1. a) Sketch the graph of the function

$$f(x, y) = \sqrt{16 - x^2 - y^2}.$$

Describe the domain and range. (3 points)

b) Compute the limit

$$\lim_{(x,y)\to(0,0)} \frac{2x}{\sqrt{x^2 + y^2}}.$$

or prove that it does not exist. (3 points)

c) Determine the set of points at which the function

$$f(x, y) = \frac{x - y}{1 + x^2 + y^2}$$

is continuous. (3 points)

2. Let $f(x, y) = \sqrt{2x + 3y}$.

a) Compute f_x and f_y. (3 points)

b) Find the slope of the surface $z = f(x, y)$ in the x-direction at the point $(5, 2)$. (3 points)

3. Let $L(x, y)$ denote the local linear approximation to

$$f(x, y) = \ln(x - 3y)$$

at the point (7,2). Use $L(x, y)$ to approximate $f(6.9, 2.06)$. Compare your answer with the exact value from the calculator. (8 points)

4. a) Use the chain rule to find $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ if

$$f(x,y) = \frac{x}{y}, \quad x = 2\sin u, \quad y = 3\cos v.$$

Express your answer in terms of u and v. (5 points)

b) Given $z = f(x,y)$. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if

$$x^2 + y^2 + z^2 = 3xyz.$$

(5 points)

5. a) Find a unit vector in the direction in which

$$f(x, y) = 3x - \ln y$$

increases most rapidly at $P(2, 4)$, and find the rate of change in that direction. (6 points)

b) Find an equation for the tangent plane to the surface

$$z = 4x^3y^2 + 2y - 2$$

at the point $P = (1, -2, 10)$. (6 points)

6. Find all the local maximum and minimum values and saddle point(s) of the function

$$f(x, y) = (1 + xy)(x + y).$$

(10 points)