Homework 2

Mutivariable Calculus

Due on Monday, February 3

11.1 Three dimensional space

1. Find an equation of the sphere that passes through the point (4,3,-1) and has center (3,8,1).

2. Describe the set of all points in 3d-space whose coordinates satisfy the inequality $(x-1)^2 + y^2 + (z+4)^2 \geq 25$.

11.2 Vectors

3. Let $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}, \mathbf{w} = \mathbf{j} + 2\mathbf{k}$. Find

   a) $\|\mathbf{v}\|$

   b) $3\mathbf{v} - 4\mathbf{w}$

   c) unit vector of $\mathbf{v} + \mathbf{w}$.

4. Let vectors $\mathbf{u} = \mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and $\mathbf{v} = a\mathbf{i} + 12\mathbf{j} + b\mathbf{k}$. Find constants a and b that make the vector $\mathbf{u}$ parallels to $\mathbf{v}$.

11.3 Dot product

5. Use dot product to find the angle that vector $\mathbf{v} = -\sqrt{3}\mathbf{i} + \mathbf{j}$ makes with the vector $\mathbf{w} = \mathbf{i}$, (positive x-axis).

6. (a) Show that (2,1,6), (4,7,9) and (11,7,-12) are the vertices of a right triangle.

   (b) Find the area of the triangle.

7. Find r so that the vector from the point $A(1,-1,3)$ to the point $B(3,0,5)$ is orthogonal to the vector from A to the point $P(r,r,r)$.

8. Given the points $P_1 = (2,1,6), P_2 = (4,7,9)$, and $P_3 = (11,7,-12)$.

   a) Find the distance from P_1 to P_2.

b) Find the vector with length 2 and has opposite direction to the vector $\overrightarrow{P_1P_2}$.

c) Show that the vector $\overrightarrow{P_1P_2}$ is orthogonal (perpendicular) to the vector $\overrightarrow{P_1P_3}$.

9. **True-False** Determine whether the statement is true or false. Explain your answer.

i) _____ If $\mathbf{v} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{w}$ and $\mathbf{v} \neq 0$, then $\mathbf{u} = \mathbf{w}$.

ii) _____ If $\mathbf{u}$ is a unit vector that is parallel to a nonzero vector $\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = \pm \|\mathbf{v}\|$.

11.4 Cross product

10. Find the following determinants:

a) $\begin{vmatrix} 4 & 5 \\ -2 & 3 \end{vmatrix}$

b) $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 0 & -3 \\ -4 & 5 & 0 \end{vmatrix}$.

11. Use a determinant to find the cross product

$$\mathbf{i} \times (\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

12. Let $\mathbf{u} = \langle 1, 2, -3 \rangle, \mathbf{v} = \langle -4, 1, 3 \rangle$.
Find $\mathbf{u} \times \mathbf{v}$ and check that it is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

13. Find two units vectors that are orthogonal to both

$$\mathbf{u} = -4\mathbf{i} + 3\mathbf{j} + \mathbf{k}, \quad \mathbf{v} = 2\mathbf{i} + 4\mathbf{k}.$$