

Homework 4

Multivariable Calculus

Due on Monday, March 9

13.1 Functions of Two Variables

1. Find the domain and range of the function

a) $f(x, y) = x^2 e^{-\sqrt{y+1}}$

b) $f(x, y) = \sqrt{x} + \sqrt{y}$

c) $f(x, y) = \sqrt{y-x} \cdot \ln(y+x)$

2. Sketch the graph of the function

(a) $f(x, y) = \sqrt{x^2 + y^2}$

(b) $f(x, y) = \sqrt{x^2 + y^2 - 1}$

(c) $f(x, y) = 3 - x^2 - y^2$.

13.2 Limits and Continuity

3. Find the limit, if it exists, or show that the limit does not exist.

a) $\lim_{(x,y) \rightarrow (5,-2)} (x^5 + 4x^3y - 5x^2)$.

b) $\lim_{(x,y) \rightarrow (3\pi, 2\pi)} xy \cos(x - 2y)$.

c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x}{\sqrt{x^2 + y^2}}$.

d) $\lim_{(x,y) \rightarrow (0,0)} \frac{2x^2y}{x^4 + y^2}$.

e) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - 16y^4}{x^2 - 4y^2}$

f) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

4. Determine the set of points at which the function is continuous.

a) $f(x, y) = \ln(x^2 + y^2 - 4)$

b) $f(x, y) = \frac{\cos(xy)}{e^x - y^2}$.

13.3 Partial Derivatives

5. Find $f_x(x, y)$ and $f_y(x, y)$ where $f(x, y) = 6e^{x^2y^3}$.
6. Let $f(x, y) = xe^{-y} + 5y$.
 - a) Compute f_x and f_y .
 - b) Find the slope of the surface $z = f(x, y)$ in the y -direction at the point $(2, 5)$.
7. A point moves along the intersection of the elliptic paraboloid $z = x^2 + 3y^2$ and the plane $y = 1$. At what rate is z changing with respect to x when the point is at $(3, 1, 12)$?
8. Find all the second partial derivatives of the function
 - a) $f(x, y) = x^4 - 3x^2y^3$.
 - b) $f(x, y) = \ln(3x + 5y)$.