Homework 5

Mutivariable Calculus

Due on Monday, March 23

13.3 Partial Derivatives

1. If $f(x, y) = 9 - 4x^2 - y^2$, find $\lim_{\Delta x \to 0} \frac{f(2 + \Delta x, 1) - f(2, 1)}{\Delta x}$.

2. Evaluate $\frac{\partial^{100}}{\partial x^{95} \partial y^2 \partial x^3}(ye^x x + \cos(x))$.

3. If $z = f(x, y)$ with the relation $xy^2z^3 + x^3y^2z + 1 = x + y + z$. Find $\frac{\partial z}{\partial x}$ at $(1, 1, 1)$.

13.4 Differentiability, Differentials, and Local Linearity

4. Find the linearization $L(x, y)$ of $f(x, y) = x\sqrt{y}$ at $(1, 4)$.

5. Use the linearization of $f(x, y) = \sqrt{x + 2y}$ to approximate $f(3.01, 2.96)$.

6. Find the differential of $z = y \cos(xy)$.

7. If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, find the value of dz.

13.5 The Chain Rule

8. Let $z = x \sin(xy)$, and suppose that $x = e^{t^2}, y = 2t$. Use the chain rule to compute $\frac{dz}{dt}$.

9. Use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

   $z = e^{xy}, \quad x = s + 2t, \quad y = s/t.$

10. Use implicit differentiation to find dy/dx of

$$\cos(x - y) = xe^y.$$

13.6 Directional Derivative and Gradient Vector

11. Find the directional derivative of the function

$$f(x, y) = \ln(x^2 + y^2)$$

    at the point $(2, 1)$ in the direction of the vector $\mathbf{u} = \langle -1, 2 \rangle$.

12. Find the maximum rate of change of $f(x, y) = y^2/x$ at the point (2,4) and the direction in which it occurs.

13. Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin xy$ at the point $(1, 0)$ has the value 1.