Class 10, Practice Problems

Mutivariable Calculus

March 9, 2020

13.5 The Chain Rule

1. Use the chain rule to find dz/dt.

   (a) $z = x^2 y + xy^2, \quad x = 2 + t^2, \quad y = 1 - t^3.$

   (b) $z = \sin x \cos y, \quad x = \pi t, \quad y = \sqrt{t}.$

2. Use the chain rule to find $\partial z/\partial s$ and $\partial z/\partial t$ where

$$z = \frac{x}{y}, \quad x = se^t, \quad y = 1 + se^{-t}.$$

3. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $x^3 + y^3 + z^3 + 6xyz = 1.$

13.6 Directional Derivatives and Gradients

1. Let $f(x, y) = 5xy^2 - 4x^3 y$, point $P = (1, 2)$ and vector $\mathbf{u} = \langle \dfrac{5}{13}, \dfrac{12}{13} \rangle.$

   (a) Find the gradient of f.

   (b) Find the gradient of f at point P.

   (c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

   (d) Find the maximum rate of change of f at point P.