April 12, 2009

Lab 10: It all adds up

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems with your teammate, and write up your solutions neatly, clearly and carefully. Both members of the team should understand and be able to explain the solutions.

Strategy for testing series

There is no set list of techniques that you should go through in order to determine convergence or divergence of an infinite series, $\sum_{n=0}^{\infty} a_n$. However, here are some general strategies that may help you decide which test might work.

(1) **Test for divergence.** Good first test: see if you can tell at a glance that $\lim_{n\to\infty} a_n = 0$. If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges by the test for divergence.

(2) **Geometric series.** If you see terms that look like c^n in the summation, you may be able to turn it into a geometric series of the form $\sum ar^n$. In this case the series diverges when $|r| \geq 1$ and converges when $|r| < 1$. (Remember that in the convergent case you have two formulas that can help you compute the sum: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ and $\sum_{n=1}^{\infty} ar^n = \frac{ar}{1-r}$).

(3) **Compare to geometric series.** If you see terms that look like c^n in the summation, but you can't turn it into a geometric series, then you may want to compare the series to a geometric series (using either the direct comparison or limit comparison test, whichever is easier).

(4) **p-series.** A series of the form $\sum \frac{1}{n^p}$ is a p-series. It converges when $p > 1$ and diverges when $p \leq 1$ (the $p = 1$ case is the harmonic series).

(5) **Compare to p-series.** If the series looks like $\sum \frac{r(n)}{q(n)}$ where $r(n)$ and $q(n)$ are polynomials, then you may want to compare the series to the p-series $\sum \frac{1}{n^p}$ where $p = (\text{degree of } q) - (\text{degree of } r)$ (using either the direct or limit comparison test, whichever is easier). This same hint

applies if $r(n)$ and $q(n)$ are roots of polynomials.

(6) **Comparison test.** Remember that the direct and limit comparison tests only apply to series $\sum a_n$ where the a_n are positive. If the terms are not positive, you may want to use the comparison tests on $\sum |a_n|$ and show absolute convergence.

(7) **Integral test.** Most of the time we can get away by using some other simpler tests. There are some series that we could not avoid using the integral test such as $\sum_{n=2}^{\infty} \dfrac{1}{n \ln n}$. Remember to check the three necessary conditions: continuous, positive and decreasing, before you apply the test.

(8) **Alternating series test.** If the series has the form $\sum(-1)^n b_n$ or $\sum(-1)^{n+1} b_n$, then try the alternating series test. If the signs are alternating, the b_n's are decreasing and $\lim_{n\to\infty} b_n = 0$, then the series converges.

(9) **Ratio test for absolute convergence.** The ratio test is useful when you see factorials, constants raised to the nth power, and the terms are the product or quotient of many different equations. Remember that the ratio test will fail for all p-series, rational and algebraic functions, and conditionally convergent series.

(10) There is not always a 'right test' and a 'wrong test' for a given series. For some series there may be several way to determine convergence or divergence. However, it may be the case that one method is easier than another.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n+1}$

Converges or diverges? ______________________

Name of test: ______________________________

(2) $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

Converges or diverges? ______________________

Name of test: ______________________________

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

Converges or diverges?____________________

Name of test: __________________________

(4) $\displaystyle\sum_{k=1}^{\infty} \frac{\sin^2 k}{k^2}$

Converges or diverges?____________________

Name of test: __________________________

(5) $\displaystyle\sum_{k=0}^{\infty} \frac{2^k k!}{(k+2)!}$

Converges or diverges? ________________

Name of test: ________________

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

Converges or diverges? ________________

Name of test: ________________

(7) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n+5}$

Converges conditionally, converges absolutely, or diverges?__________

Name(s) of test(s): ____________________

(8) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

Converges conditionally, converges absolutely, or diverges?__________

Name(s) of test(s): ____________________