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Calculus II : snoitcnuf esrevni

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems with your teammate(s), and write up your solutions neatly, clearly, and carefully. All members of the team should understand and be able to explain the solutions.

I. Graphing to find inverses:

Graphing a function is a pretty good way to check to see if it has an inverse. It is not perfect since passing or not passing the H.L.T. can be hard to tell from a graph (even a computer-generated graph).

1. Consider the function
$$f(x) = \sqrt{x^3 + x^2 + x + 1}$$

   (a) What problem(s) do you experience with only using the equation to check for an inverse?

   (b) Using Grapher, plot the graph of $f(x)$. Does it look like it has an inverse? Explain.

   (c) If $f(x)$ doesn't seem to have an inverse, find a horizontal line which intersects the graph in more than one place. If it does seem to have an inverse, use Grapher to plot the inverse. (Note: Grapher doesn't require an equation to be in the form of $y =$.)

II. Using derivatives to find the existence of an inverse:

Derivatives (one of the main ideas of Calculus I) can be used to determine whether or not a function has an inverse.

1. If the graph of a function is always increasing or always decreasing, what must be true if any horizontal line is placed somewhere on the graph?

2. If a function is always increasing, what must be true of its derivative? What if the function is always decreasing?

3. Consider the function
$$g(x) = x^5 + 2x^3 + 4x + 8$$

   (a) What problem(s) do you experience with only using the equation to check for an inverse?

   (b) Find the derivative, $g'(x)$.

   (c) Using Grapher, plot the graph of $g'(x)$. Where is the graph of $g'(x)$ located in the cartesion plane? Does this location tell you anything about the graph of the original function?

(d) What can you conclude about the existence of an inverse for $g(x)$.

4. Consider the function
$$p(x) = x^3 + 5x + 2.74$$

(a) Without using Grapher, find the derivative of $p(x)$. Can you make the argument, based solely on the derivative equation, that $p(x)$ has or doesn't have an inverse. Explain.

III. Every function has an inverse (sort of).

Every function doesn't have an inverse. Most functions do not pass the H.L.T. However, if we restrict the domain of the function, we could create a piece of the original function that has an inverse.

1. Using Grapher, plot $g(x) = x^3 - 8x + 3$. Find a domain in the form of a closed interval over which $g(x)$ has an inverse.

2. Find a domain over which $g(x)$ has an inverse which is as large as possible. (Note: This domain could be the union of intervals.)

3. Next, consider $h(x) = \cos x$. Using Grapher, plot $h(x)$. Find the largest interval possible, which includes 0, over which $h(x)$ has an inverse.