February 2, 2009

Calculus II : Two Steps Forward, One Step Back

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems with your teammate(s), and write up your solutions neatly, clearly, and carefully. All members of the team should understand and be able to explain the solutions.

I. Consider the function

$$y = \ln\left[\frac{(x^2+7)(x-4)^3}{(x-1)}\right].$$

Its derivative is quite simple IF we first apply the properties of logarithms discussed in class. Namely,

i. $\ln xy = \ln x + \ln y$ ii. $\ln x/y = \ln x - \ln y$ iii. $\ln(x^r) = r \ln x$

a. Simplify the function using the properties.

b. Find the derivative of the (simplified) function.

It was nice to see the "ln" in the previous problem. Otherwise, we wouldn't have been able to simplify our function before taking the derivative ... or could we have ... ????

II. Consider

$$y = \frac{x^5(x^3+9)^2}{(x+6)}$$

which is similar to the previous problem but there is no "ln." That can be fixed by taking the logarithm of the function.

a. Take the natural logarithm of BOTH sides of the equation.

b. Using natural logarithmic properties, expand the right side of the equation.

c. Next, find the derivative of the equation with respect to x: you need to take the derivative of the right side AND the left side. Note that the left side of the equation will need implicit differentiation. (For example, the derivative of $\cos(y)$ is $-\sin(y) \cdot y'$.)

d. Finally, make sure that you solve for $\frac{dy}{dx}$ and that it is written as a function of x (i.e., no y's are part of the dervivative).

This technique, logarithmic differentiation, made a difficult differentiation problem easier!!

III) Consider the function $y = x^x$. We have NO derivative rule to use on this function. However, we can use logarithmic differentiation. Find y'.

IV) Again, use logarithmic differentiation to find the derivative (with respect to x) of

$$y = 7^x$$

Remember: A function like $y = \ln\left(\frac{x^4(x^2-9x)}{\cos x}\right)$ doesn't need logarithmic differentiation since you are already taking the natural logarithm of the right side.

V. Finally, use the technique of logarithmic differentiation to find the derivative of : $y = e^x$.

VI. Suppose that Bill Gates' net worth (in billions of dollars) can be estimated by how many months n that he has been alive using the following equation:

$$\text{Bill Gates' net worth } = \left(1 + \frac{1}{n}\right)^n.$$

a. Using Calculator, find out how much he was worth at age 20.

Net worth at age 20 =

b. How much will he be worth at 50 years old?

Net worth at age 50 =

c. How much will he be worth at 100 years old?

Net worth at age 100 =

d. If this continues after his death, estimate how much his estate will he be worth in the far distant future. (Does this number look familiar?)