February 23, 2009

Lab 5: Integration Practice

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems and write up your solutions neatly, clearly and carefully. All members of the team should understand and be able to explain the solutions.

Strategy for Integration

Integration is much more difficult than differentiation. There are some strategies, however, that can help you decide which technique might work.

1. Simplify the integrand: use algebra or a trigonometric identity to rewrite the integrand in a form which you will be able to integrate.

   Example: $\displaystyle\int \frac{x^5 - x^3 - 8}{x} dx = \int x^4 - x^2 - \frac{8}{x} dx.$

2. Look for an obvious substitution: try to find some function u whose derivative du also appears in the integrand.

   Example: $\displaystyle\int \frac{x}{x^2 - 3} dx$ can be solved by the substitution $u = x^2 - 3$.

3. Classify the integrand by its form:

   (a) Integration by parts: if your integrand is the product of two functions then integration by parts may be useful.

   (b) Trigonometric functions: If the integrand is a product of powers of sine and cosine, try a trig. identity.

   (c) If you see $\pm x^2 \pm a^2$ in the integrand, try a trig. substitution.

4. Try again. If you haven't completed the integral, you may need to use a combination of methods. See if you can find a different substitution to try. Try to use algebra or trigonometric identities to manipulate the integrand into a simpler form.

5. There is not always a 'right way' and a 'wrong way' to approach an integral. For some integrals there may be several way to find the solution. However, it may be the case that one method is easier than another.

Remember: You can always check if your answer! The derivative of your answer should be the original integrand.

Part I Integration Practice:

1. $\displaystyle\int \frac{e^{\tan^{-1} y}}{1+y^2}\, dy$

2. $\displaystyle\int \sqrt{z}(z+\sqrt[3]{z})\, dz$

3. $\int x \sin 3x \; dx$

4. $\int \sin \sqrt{x} \; dx$

5. $\int \sin^2 x \cos^3 x \, dx$

6. $\int \frac{1}{x^2\sqrt{16x^2 - 9}} \, dx$

7. $\int e^{x+e^x}\ dx$

8. $\int x^4 \ln x\ dx$

9. Consider the integral $\int \frac{x}{9-x^2}\, dx$.

(a) Compute the integral using u-substitution.

(b) Compute the integral using trig. substitution.

(c) Explain why the two answers are the same.

Part II The Difficult Case: $\int \tan^n x \sec^m x \; dx$, when n is even and m is odd.

During the class last week, we discussed $\int \sec x \; dx$ and $\int \sec^3 x \; dx$.

We use u-substitution to find $\int \sec x \; dx$:
$\int \sec x \; dx = ln|\sec x + \tan x| + C$.

We use integration by parts to find $\int \sec^3 x \; dx$:
$\int \sec^3 x \; dx = \frac{1}{2}(\sec x \tan x + ln|\sec x + \tan x|) + C$.

This exercise will guide you to do the integration of this form with higher values of n and m.

1. $\int \sec^m x \; dx$, m is odd

   Using integration by parts with $u = \sec^{m-2} x$ and $dv = \sec^2 x \; dx$.

   Try $\int \sec^5 x \; dx$

2. Once we find $\int \sec^m x \; dx$, m is odd, we can evaluate all the integrations of the form $\int \tan^n x \sec^m x \; dx$, when n is even and m is odd by using trigonometric identities to write things in terms of $\sec x$.

   (a) $\int \tan^2 x \sec^3 x \; dx$

   (b) $\int \tan^4 x \sec x \; dx$