

Lab 6: Partial Fractions

TEAM MEMBERS

INSTRUCTIONS: Work the following problems and write up your solutions neatly, clearly and carefully. All members of the team should understand and be able to explain the solutions.

PART 1: Fun with Rational Functions - A Warm-up Problem

Here is a “new” integration problem to set the stage for today’s lab.

1. Find $\int \frac{4x + 3}{(x^2 + 25)^2} dx$. (Hint: Split it up into two integrals. That is, put each piece of the numerator over the whole denominator. One part should lend itself to a u -substitution and the other part to a trig substitution (even though there is no square root).)

2. Describe (briefly) what would change (in terms of the integration techniques that you used) in your work above if the exponent in the denominator of the problem would change.

Have this page graded
before proceeding.

PART 2: More Fun with Rational Functions

When integrating rational functions, the difficulty is not in the final integrations but in factoring the denominator and then finding the “anti-common denominator.” Maple has a command that facilitates this process. The command is:

$$\text{convert}(f, \text{parfrac});$$

where f is the rational function. Let’s try it.

1. Use Maple’s `convert` command on the following rational functions.

(a) $\frac{17x + 14}{3x^2 + 7x + 2}$

(b) $\frac{-11 + 28x + 30x^2 + 7x^3}{x^4 + 5x^3 + 6x^2 - 4x - 8}$

(c) $\frac{x^4 - 2x^3 - 15x^2 + 5x - 24}{x - 5}$

2. Integrate (by hand) the rational functions in question 1 using the results of Maple’s `convert` command.

(a) $\int \frac{17x + 14}{3x^2 + 7x + 2} dx =$

$$(b) \int \frac{-11 + 28x + 30x^2 + 7x^3}{x^4 + 5x^3 + 6x^2 - 4x - 8} dx =$$

$$(c) \int \frac{x^4 - 2x^3 - 15x^2 + 5x - 24}{x - 5} dx =$$

PART 3: Even More Fun with Rational Functions

In class, the integrals of the form

$$\int \frac{A}{(ax + b)^n} dx, \text{ where } n = 1 \text{ or } 2 \text{ or } 3 \text{ or } \dots$$

could each be solved using u -substitution. The rational functions below will also produce some of these same integrals but will also produce some new ones. The new ones occur when the denominator does NOT factor into linear terms.

1. Use Maple's convert command on the following two rational functions.

$$(a) \frac{18 + 11x + 3x^2}{24 + 6x + 4x^2 + x^3} \quad (b) \frac{75 - 37x + 38x^2 - 4x^3 + 3x^4}{x^5 - 2x^4 + 18x^3 - 36x^2 + 81x - 162}$$

2. Integrate the rational functions in question 1 using the results of Maple's convert command and PART 1 of *this* lab.

$$(a) \int \frac{18 + 11x + 3x^2}{24 + 6x + 4x^2 + x^3} dx =$$

$$(b) \int \frac{75 - 37x + 38x^2 - 4x^3 + 3x^4}{x^5 - 2x^4 + 18x^3 - 36x^2 + 81x - 162} dx =$$