March 30, 2009

Lab 8: Lists, Carpets, and Bouncing balls

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems and write up your solutions neatly, clearly and carefully. All members of the team should understand and be able to explain the solutions.

1. Fill in the blanks in the following table.

5 terms of the sequence	**Formula**	**Limit as $n \to \infty$**
(i) $1, 2, 3, 4, 5, \ldots$	$\{n\}_{n=1}^{\infty}$	The sequence diverges to infinity.
(ii) ______	$\{(-1)^{n+1}2^{-n}\}_{n=0}^{\infty}$	______
(iii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$	$\{$ ______ $\}_{n=5}^{\infty}$	______
(iv) ______	$\{\sin(n\pi)\}_{n=0}^{\infty}$	______
(v) ______	$\left\{\frac{n^2+1}{n+1}\right\}_{n=0}^{\infty}$	______
(vi) $\frac{e^3}{9}, \frac{e^4}{16}, \frac{e^5}{25}, \frac{e^6}{36}, \frac{e^7}{49}, \ldots$	______	______
(vii) $\frac{1}{0!}, -\frac{x^2}{2!}, \frac{x^4}{4!}, -\frac{x^6}{6!}, \frac{x^8}{8!}, \ldots$	______	N/A
(viii) ______	______	Converges to 5
(ix) ______	______	Oscillates between 3 and 4

2. Easy as one, two, three. Find the next element of the given sequence and state the pattern.

   (a) 0, 1, 3, 7, 15, 31, 63,. . .

   (b) 2,3,5,7,11,. . .

   (c) 2,3,6,18,108,. . .

   (d) 2,7,1,8,2,. . .
   (**Hint:** Do you know all the numerical values of all the famous mathematical constants?)

   (e) 2, 3, 3, 5, 10, 13, 39, 43, 172, 177,. . .

Remark: In general one needs a formula to make a sequence. One can recognize what one thinks is a pattern, but in fact the sequence may have a different pattern or even no pattern at all.

3. Suppose that you start with a 1×1 piece of paper. A **Sierpinski carpet** is to be made from your paper. It is constructed by removing the center one-ninth of the square, then removing the center one-ninths of the eight smaller remaining squares, and so on. The first three steps of the construction are shown.

(a) If we do this procedure an infinite number of times, will any paper left?

(b) Find the sum of the areas of the removed squares.

(c) What is the area of the remaining paper?

This interesting and beautiful shape is called a *fractal*. It has many strange and interesting properties such as this one.

4. A certain ball has the property that **each time** it falls from a certain height onto a hard, level surface, it rebounds up $100r\%$ of the height that it just fell where $0 \leq r < 1$.

   Suppose that the ball is released from a height of H feet above ground level. Assuming that the ball bounces indefinitely, find the total distance the ball travels.