April 6, 2009

Lab 9: Fun with Series

TEAM MEMBERS

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INSTRUCTIONS: Work the following problems and write up your solutions neatly, clearly and carefully. All members of the team should understand and be able to explain the solutions.

I. Geometric Series come in many forms. For example, the following are geometric series:

$$\sum_{n=0}^{\infty} 3\left(\frac{2}{7}\right)^n \qquad \sum_{n=0}^{\infty} \frac{3\left(\frac{2}{3}\right)^{n+7}}{5^{n-3}} \qquad \sum_{n=3}^{\infty} \frac{4 \cdot 3^{n+7} \cdot 2^{-9n+4}}{3 \cdot 2^{-n-1} \cdot 11^{2n+1}}$$

However, all of them can be written as

$$\sum_{n=}^{\infty} a \cdot r^n.$$

Putting them in this simple form is accomplished by using algebraic properties of "exponential-type functions." They are:

$$a^{i+j} = a^i a^j \qquad a^n = \frac{1}{a^{-n}} \qquad (a^i)^j = a^{ij} \qquad \frac{a^i}{b^i} = \left(\frac{a}{b}\right)^i \qquad a^i b^i = (ab)^i$$

For example,

$$3\left(\frac{1}{7}\right)^{2n+3} = 3\left(\frac{1}{7}\right)^{2n}\left(\frac{1}{7}\right)^{3} = 3\left(\left(\frac{1}{7}\right)^{2}\right)^{n}\left(\frac{1}{7}\right)^{3} = 3\left(\frac{1}{49}\right)^{n}\left(\frac{1}{343}\right) = \frac{3}{343}\left(\frac{1}{49}\right)^{n}.$$

It is then easy to see that the first term of the series $\sum_{n=2}^{\infty} 3\left(\frac{1}{7}\right)^{2n+3}$ is $\frac{3}{343}\left(\frac{1}{49}\right)^2$ and $r = \frac{1}{49}$.

a). Put the following series in the form $\sum_{n=3}^{\infty} a \cdot r^n$ and see if it converges or diverges.

$$\sum_{n=3}^{\infty} \frac{4 \cdot 3^{n+7} \cdot 2^{-9n+4}}{3 \cdot 2^{-n-1} \cdot 11^{2n+1}} =$$

II. Using the Integral Test, show divergence of the general harmonic series: $\displaystyle\sum_{n=1}^{\infty} \frac{1}{an+b}$, where a and b are constants.

III. a) How do $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ compare? That is, which one is bigger? Explain.

b). Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (it is a p-series with $p = 2 > 1$) and considering your answer to part III (a), what can you say about the convergence or divergence of $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$? Explain.

c) To convince yourself that your answer to part III (b) is correct, use the Integral Test on the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

IV. **a)** Suppose $\sum_{n=1}^{\infty} a_n$ converges. Then we know $\lim_{n\to\infty} a_n = 0$. (If it didn't equal 0, then it would have to diverge by the Test for Divergence.) Now consider $\sum_{n=1}^{\infty} \frac{1}{a_n}$. Use the Test for Divergence on this series. What can you conclude about this series?

V. An infinite series is a sum of numbers (e.g., $1 + \frac{1}{3} + \frac{11}{29} + \cdots$). So, an infinite series written in summation notation does not contain any variables. For example, in $\sum_{n=1}^{\infty} \frac{n^5}{2 \cdot 3^n}$, n is NOT a variable. However, $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ does contain a variable.

$$\sum_{n=1}^{\infty} \frac{x^n}{3^n} = \frac{x^1}{3^1} + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \frac{x^4}{3^4} + \cdots$$

In this case, instead of an infinite sum, we have an infinite polynomial!

a). For what values of x does this series converge? (Hint: If you plug in a number for x, what kind of series will it be?)