Chapter 8
Confidence Intervals

1 Confidence Intervals for a Population Mean

Assumption: $\bar{x}$ has a normal distribution

Condition

- Original population distribution is normal or
- Sample sizes, n is large (≥ 30) (Central Limit theorem)

$$(1-\alpha)100\% \text{ confidence interval} = \left[\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right]$$

Remark: If σ is unknown,

$$(1-\alpha)100\% \text{ confidence interval} = \left[\bar{x} - z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{s}{\sqrt{n}}\right]$$

under the additional condition that $n \geq 30$.

2 Confidence Intervals for a Population Proportion

Assumption: $\hat{p}$ has a normal distribution

Condition

- $np \geq 5$ and $n(1-p) \geq 5$

$$(1-\alpha)100\% \text{ confidence interval} = \left[\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

3 Sample Size Determination

The smallest sample size n for confidence interval to contain in the error bound.

- Population mean: $n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{E}\right)^2$
- Population proportion: $n = p(1-p)\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2$