First Name_______________ Last Name_____________________ ID. _________

Mahidol University International College

Final Examination

Trimester 2/ 2018-2019

Course/Code: Essential Statistics/ ICGN 103

Instructor : Aj. Thotsaporn THANATIPANONDA Section 1

Aj. Thotsaporn THANATIPANONDA Section 2

Date:	Saturday, 6 April 2019
Time:	12:00 - 13:50
Total pages:	Exam 8 pages, scratch paper (last 2 pages)

Directions

1. There are 12 questions. Total score is 32 points (scale to 35 % of total grade).
2. Students are allowed to bring ONLY one-side A4 paper of the examiners handwritten information and calculator(s) into the examination room.
3. Show a reasonable amount of work.
4. You may tear apart the scratch paper. We will NOT grade any information in the scratch paper.
5. Students found cheating during the examination will be penalized according to the university's examination policy.

1. True/False questions (1 point each)

   a) __________ The probability that a standard normal random variable will be between 0 and 1.5 is 0.4332.

   b) __________ The Central Limit Theorem states that the sampling distribution of the sample mean ($\bar{x}$) is approximately normal if the sample size (that is used to calculate $\bar{x}$) is large enough ($n \geq 30$).

   c) __________ A very wide confidence interval gives the message that there is a great deal of uncertainty concerning the value of what we are estimating.

   d) __________ When conducting a hypothesis test about a sample mean, $\bar{x}$, other relevant factors held constant, increasing the level of significance α from 0.01 to 0.05 will reduce the chance of making a Type I error.

2. (3 points) The resistance of an electrical component follows a probability density function given by

$$f(x) = \begin{cases} \frac{x}{4}, & 1 < x < 3; \\ 0, & otherwise. \end{cases}$$

   a) Sketch the function and verify that the total area under the curve of $f(x)$ is equal to 1.

   b) Find the probability that the resistance is less than 2.

3. (1 point) Which of the following statements is NOT a property of the normal probability distribution?

   A) The shape of any individual normal curve depends on its specific mean and standard deviation.

   B) The highest point is over the mean (as well as the median and mode).

   C) The curve is symmetrical about its mean. The left and right halves of the curve are mirror images of each other.

   D) $P(X = \mu)$, where μ is the mean, is always equal to 0.5.

4. (2 points) The average grade for a final exam is 84, and the standard deviation is 7. Suppose that 12% of the class are given As, and the grades are curved to follow a normal distribution. What is the lowest possible score of a student who gets an A?

5. (2 points) On test A, the mean score is 100 with a standard deviation of 15. On test B, the mean score is 400 with a standard deviation of 50. You scored 115 on test A and 425 on test B.

   Assume the distributions of scores of test A and test B follow normal distribution. By comparing the percentiles, which test did you do better?

6. (3 points) Suppose it is known that the mean blood glucose (mg/mL), μ, in a group of diabetic rats treated with a drug is 1.85 mg/mL with $\sigma = 0.4$ mg/mL. Given a sample of 36 diabetic rats and are treated with drug. Approximate the probability that the sample mean will be less than 1.9 mg/mL.

7. (2 points) **Give short answer.**
   If many samples are taken and a 90% confidence interval for, population proportion, p is constructed for each sample, then what percents of the intervals one can expect not to cover the unknown p?

8. (3 points) In a randomly selected group of 650 automobile deaths, 180 were alcohol related. Construct a 95 percent confidence interval for the true proportion of all automobile accidents caused by alcohol.

9. (3 points) The college president asks the statistics teacher to estimate the average age of the students at their college. How large a sample is necessary? The statistics teacher would like to be 99% confident that the estimate should be accurate within 1 year. From a previous study, the (population) standard deviation of the ages is known to be 2 years.

10. (2 points) An inspector has to choose between certifying a building as safe or saying that the building is not safe. There are two hypotheses:
    H_0 : Building is safe
    H_a : Building is not safe

    State Type I and Type II error of this scenario. Which type of error has more serious consequence?

11. (4 points) A manufacturer claims that the thickness of the spearmint gum it produces is 7.5 one-hundredths of an inch. A quality control specialist regularly checks this claim. On one production run, he took a random sample of $n = 30$ pieces of gum and measured their thickness. He obtained:

7.65, 7.60, 7.65, 7.70, 7.55, 7.55, 7.40, 7.40, 7.50, 7.50, 7.70, 7.55, 7.55, 7.40, 7.40, 7.55, 7.40, 7.40, 7.50, 7.50, 7.70, 7.55, 7.55, 7.40, 7.40, 7.40, 7.40, 7.50, 7.50, 7.70.

The mean of this data is 7.52 with standard deviation of 0.1088. Conduct the hypothesis testing to test the thickness of the spearmint gum with significance level $\alpha = 0.01$.

a) Set up null and alternative hypotheses.

b) Determine the test statistic.

c) Test the hypothesis at $\alpha = 0.01$. What is your conclusion?

12. (3 points) Among patients with lung cancer, usually 90% or more die within three years. As a result of new forms of treatment, it is felt that this rate has been reduced. In a recent study of $n = 150$ lung cancer patients, 128 died within three years. The null and alternative hypotheses are:

$$H_0 : p = 0.9 \quad \text{and} \quad H_a : p < 0.9$$

In this case, the test statistic $z =$ -1.91

a) Find the p-value.

b) Use p-value approach to test whether there is sufficient evidence at the $\alpha = 0.05$ level, say, to conclude that the death rate due to lung cancer has been reduced?

Answers

1. a) True, $P(0 \le z \le 1.5) = P(z \le 1.5) - P(z \le 0) = 0.9332 - 0.5 = 0.4332$

   b) True

   c) True, remember what I said during the class, the smaller the confidence interval, the less uncertainty which is better!

   d) False, Increasing α will increase the chance of rejecting H_0. Hence also **increase** the chance of making a Type I error (reject H_0 when it is true).

   Problems number 2,3,4,5 are problems from chapter 6 and will not be in the final.

6. $P(\bar{x} < 1.9) = P(z < \dfrac{1.9 - 1.85}{0.4/\sqrt{36}}) = P(z < 0.75) = 0.7734.$

7. We will miss the population proportion p about 10% of the time.

8. $\hat{p} = \dfrac{180}{650}$.

   The 95% CI $= \left[\hat{p} - z\sqrt{\dfrac{\hat{p}\hat{q}}{n}}, \hat{p} + z\sqrt{\dfrac{\hat{p}\hat{q}}{n}}\right]$

   $= \left[0.2769 - 1.96\sqrt{\dfrac{(0.2769)(0.7231)}{650}}, 0.2769 + 1.96\sqrt{\dfrac{(0.2769)(0.7231)}{650}}\right]$

   $= [0.2425, 0.3113]$.

9. The sample size $n = \left(\dfrac{z\sigma}{E}\right)^2 = \left(\dfrac{2.575 \cdot 2}{1}\right)^2 = 26.5225.$

   Therefore we need at least 27 samples

10. Type I error: an inspector says the building is not safe when it is actually safe. Type II error: an inspector says the building is safe when it is not safe. Of course Type II error has more serious consequence as the building can collapse and kill people.

11. $\mu =$ mean thickness of the gum.

    a) $H_0 : \mu = 7.5 \;\; VS \;\; H_a : \mu \neq 7.5.$

    b) Test statistic $z = \dfrac{7.52 - 7.5}{0.1088/\sqrt{30}} = 1.0068.$

    c) The critical value at $\alpha = 0.01$ (two sided test) is ± 2.575. The test statistic does not fall in the rejection region. We do not reject H_0. The assumption that $\mu = 7.5$ still holds true.

12. a) p-value $= P(z < -1.91) = 0.0281.$

    b) Since the p-value$=0.0281 < 0.05 = \alpha$, we reject H_0. We conclude H_a that the death rate due to lung cancer has been reduced.