On the Minimum Number of Monochromatic Generalized Schur Triples.

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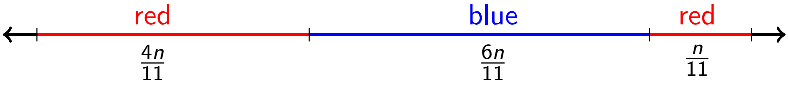
Original Problem

In 1996, Ronald Graham asked the question about the minimum number of monochromatic triples (x, y, z) satisfies equation $x + y = z$ of any 2-coloring of the interval $[1, n]$.

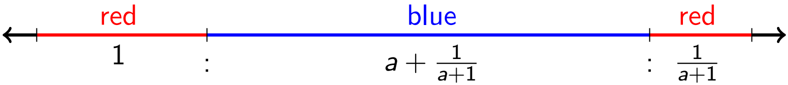
The answer was confirmed four times by Robertson and Doron Zeilberger(1998), Schoen(1999), Datskovsky(2003) and Thanatipanonda(2009) to be $\frac{n^2}{22} + O(n)$.



The Optimal Coloring for $x + y = z$

Recent Work

Recently Wong and myself showed that the minimum numbers of monochromatic triples of the form $\{x, y, x + ay\}$, $a \geq 2$, of any 2-coloring of the interval $[1, n]$, are $\frac{n^2}{2a(a^2+2a+3)} + \mathcal{O}(n)$.



The Optimal Coloring for $x + ay = z$

Conjectures

We present conjectures on variations of Graham's original problem. Denote R, B, G to be the colors red, blue, and green respectively.

1. Equation: $ax + by = az$ where a, b are integers.
   1. Case 1: $a > b \geq 2, \quad gcd(a, b) = 1$
      The coloring that gives the minimum number of monochromatic solutions over any 2-coloring of $[1, n]$ is
      $$\left[(R^{a-1}, B)^{\frac{n}{a}}\right].$$
   2. Case 2: $b > a \geq 2, \quad gcd(a, b) = 1$
      The coloring that gives the minimum number of monochromatic solutions over any 2-coloring of $[1, n]$ is
      $$\left[(R^{a-1}, B)^{\frac{n}{b}}, \quad R^{(\frac{b-a}{b})n}\right].$$

Second Conjecture

2. Equation: $x + y + w = z$
   The coloring that gives the minimum number of monochromatic solutions over any 2-coloring of $[1, n]$ is

$$\left[R^{\frac{3(10-\sqrt{3})n}{97}}, B^{\frac{(6+\sqrt{3})(10-\sqrt{3})n}{97}}, R^{\frac{(10-\sqrt{3})n}{97}} \right],$$

   with the number of monochromatic solutions to be

$$\frac{n^3}{12(10+\sqrt{3})^2} + \mathcal{O}(n^2).$$

Third Conjecture

3. Equation: $x + y = z$
   The coloring that gives the maximum number of rainbow solutions over any 3-coloring of $[1, n]$ is

$$\left[(R, B)^{\frac{n}{5}}, (G, B)^{\frac{3n}{10}}\right],$$

with the number of rainbow solutions to be

$$\frac{n(n+1)}{10}.$$