

GRAPH THEORY: CLASS 13

- 1) Only one graph of order 5 has the property that the addition of any edge produces an Eulerian graph. What is it?

- 2) Find an Eulerian trail of the graph below.

- 3) Give an example of a graph G that is
 - a) Eulerian but not Hamiltonian.
 - b) Hamiltonian but not Eulerian.
 - c) Hamiltonian and has an Eulerian trail but is not Eulerian.
 - d) neither Eulerian nor Hamiltonian, but has an Euler trail.

- 4) **Puzzle:** Termite and three dimension 27 Cubes.

Imagine a large cube formed by gluing together 27 smaller wooden cubes of uniform size as shown. A termite starts at the center of one of the face and bores a path that takes him once through every cube. His movement is always parallel to a side of the cube, never diagonal.

Is it possible for termite to bore through each of the 26 outside cube once and only once, then finish his trip by entering the central of the cube for the first time? (Hamiltonian path). If possible, show an example; if impossible, prove it!

This problem is picked from a very famous puzzle book by Martin Gardner.