

# Oral Exam

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## Enumerative Combinatorics

**Basic enumeration:** recurrence relations, inclusion-exclusion principle, permutation statistics, Stirling numbers, Bell numbers, calculus of finite differences.

**Generating functions:** formal power series, formal exponential generating functions, Dirichlet series, moments of distributions, cycle index of the symmetric group, snake oil method, Möbius inversion, Lagrange inversion.

**The five basic algorithms for hypergeometric identities:** Mary Celine Fasenmyer's algorithm, Gosper's algorithm, Zeilberger's algorithm, Petkovšek's algorithm, the WZ phenomena, dual and companion identities.

**Rational generating functions:** rational power series in one variable, transfer-matrix method, factorization in free monoids.

**Computer algebra:** Maple programming.

## References

Richard Stanley; *Enumerative Combinatorics, Vol. 1*, Chapters 1,4.

Herbert Wilf; *Generatingfunctionology*, Chapters 1,2,4.

Marko Petkovšek, Herbert Wilf, Doron Zeilberger; *A=B*, Chapters 1-8.

## Special Functions

**The gamma and beta functions:** definition, Euler's reflection formula, the Hurwitz and Riemann zeta functions, Stirling's asymptotic formula, Gauss's multiplication formula for the gamma function, Bohr-Mollerup theorem, probabilistic evaluation of the beta function.

**Hypergeometric functions:** definition, Euler's integral representation, Gauss formula, contiguous relation, dilogarithms, binomial sums, Dougall's bilateral sum.

**General Theory of Orthogonal polynomials:** three-term recurrences, Gauss quadrature, zeros of orthogonal polynomials, continued fractions, kernel polynomials, Parseval's formula.

**Classical and Special Orthogonal polynomials:** Hermite polynomials, Laguerre polynomials, Chebyshev polynomials, Jacobi polynomials, An extension of the Ultraspherical Polynomials, Wilson polynomials

**q-Series:** q-Binomial theorem, Jacobi triple product formula, Ramanujan's Summation Formula, Basic Hypergeometric series, q-Ultraspherical polynomials.

**Askey Scheme:** hypergeometric orthogonal polynomials, Askey-Wilson, Continuous dual Hahn, Jacobi, Meixner, Laguerre, Hermite and their q-analogs.

**Quasideterminants:** definition, general properties, basic identities.

## References

George Andrews, Richard Askey, Ranjan Roy; *Special Functions*, Chapters 1,2,5,6,10.

I.M. Gelfand, V. Retakh; *Quasideterminants*, *Selecta Mathematica* 3 (1997)

## Graph Theory

**Basic graph theory:** trees, bipartite graphs, Eulerian tour, path and cycles.

**Matchings:** König's theorem, Hall's theorem, Tutte's theorem, path covers and Dilworth's theorem.

**Connectivity:** structure of 2-connected and 3-connected graphs, Menger's theorem, Mader's theorem, linking pairs of vertices.

**Planar graphs:** Euler's formula, Kuratowski's theorem, planar duality.

**Random graphs:** Erdős's theorem, probabilistic method, threshold functions, second moments.

## References

Reinhard Diestel; *Graph Theory*, Chapters 1-4, 11.