

NUMBER THEORY: HOMEWORK 2

Homework due on Tuesday September 16.

1. PROBLEMS

1) Find the solution of the recurrence relation

$$a_n = 3a_{n-1} + 4a_{n-2} - 12a_{n-3}, \text{ where } a_0 = 2, a_1 = 5, \text{ and } a_2 = 13.$$

2) Define Lucas numbers by

$$L_n = L_{n-1} + L_{n-2}, \quad n \geq 3 \text{ with } L_1 = 1 \text{ and } L_2 = 3.$$

Use Maple Program to answer questions below:

- a) Find the closed form formula of L_n .
- b) Solve the identity $L_n^2 - L_{n+1}L_{n-1} = 5(-1)^n$ for all $n \geq 2$.
- c) Solve the identity $f_{2n} = f_n L_n$ for all $n \geq 1$ with f_n is the n^{th} Fibonacci numbers.

3) Find all solutions of the linear diophantine equations

$$25x + 95y = 970.$$

4) How many ways can change be made for five dollars using only dimes and quarters.

(Hint: use the technique to solve diophantine equation, but now consider only positive solutions.)

Also do the problems below

Section 2.3) 6,9

Section 3.5) 44,46