NUMBER THEORY: HOMEWORK 4

Homework due on Tuesday October 7.

1. Problems

1) Proof that if $2^n + 1$ is prime ≥ 5 then $n = 2^k$ for some positive integer k.
(There was a typo in the last homework. This is a corrected version.)

2) In 500 B.C. the Chinese seem to have known that $2^p \equiv 2 \bmod p$ for each prime p. They also assumed that if $2^n \equiv 2 \bmod n$ then n is prime.
a) Show that 341 is not a prime.
b) Show that $2^{10} \equiv 1 \pmod{341}$.
c) Show that $2^{341} \equiv 2 \pmod{341}$.

3) Let m be a positive integer. Assume that $x \equiv y \bmod m$. Show:
a) $3x + 2 \equiv 3y + 2 \bmod m$
b) $x^3 - x \equiv y^3 - y \bmod m$
c) $P(x) \equiv P(y) \bmod m$ for any polynomial P.

4) Show that $2222^{5555} + 5555^{2222}$ is divisible by 7.

5) Find integers x such that
a) $5x \equiv 4 \bmod 3$
b) $7x \equiv 6 \bmod 5$.

6) In this problem, we implement the Fermat Factorization method on Maple program.

Input: a positive integer n.
Output: a factor of n obtained from the Fermat Factorization method.

Date: Tuesday, September 30, 2008.

Below is an example of the program:

$Fermat := proc(n) \;\; local \;\; a, b;$

$a := ceil(sqrt(n));$

$while \;\; type(sqrt(a^2 - n), integer) = false \;\; do$
$a := a + 1;$
$od :$

$b := sqrt(a^2 - n);$
$return(a + b, a - b);$

$end :$

Once you're done, print out your code and the Maple worksheet with the answer to the following inputs:

a) $n = 3523$
b) $n = 2342409$
c) $n = 120938091$
d) $n = 32804989$.

Also do the following problems from the book:

problem 22 page 150.
problem 8a) and 8b) page 157.
problem 4a) page 164.