NUMBER THEORY: HOMEWORK 6

Homework due on Tuesday October 28.

1. Problems

1) Given $gcd(b, n) = 1$ and a and c are positive integers.

Prove: if $p \mid (b^n + 1)$ then either
i) $p \mid (b^d + 1)$ for some proper divisor d of n for which n/d is odd, or
ii) $p \equiv 1 \bmod 2n$.

(**Note:** This is one of the harder problem. Feel free to ask me for a hint.)

2) Use the result in problem 1 to conclude that
if $p \mid 2^{2^n} + 1$ then $p \equiv 1 \bmod 2^{n+1}$.

Double check the above statement with the actual factor of Fermat number when $n = 5, 6$ and 7.

3) Let let n be an odd number > 2.
Show that n divides at least one of the elements in the set
$\{2^2 - 1, 2^3 - 1, 2^4 - 1, ..., 2^{n-1} - 1\}$.
(Extra point if you can prove without using Euler's theorem.)

4) Let n be a pseudoprime to the base b. Show that n is also a pseudoprime to the base $-b$ and to the base b^{-1}.

5) Write a brute force program in Maple to find all the Carmichael numbers ≤ 5000. You can download outline of the program from my web site.

Also do problem 10 page 236 from the book.

Date: Tuesday, October 21, 2008.