

## NUMBER THEORY: HOMEWORK 8

Homework due on Tuesday November 25.

### 1. PROBLEMS

1) Show explicitly the bijection match between  $p(n | \text{ odd parts})$  and  $p(n | \text{ distinct parts})$  for every  $n$  between 1 and 10.

- a) Using merging-splitting procedure we discussed in class.
- b) Using Sylvester's algorithm.

2) Show that  $p(n | \text{ distinct parts and each part } \equiv \pm 1 \pmod{3}) = p(n | \text{ parts are } \pm 1 \pmod{6})$ .

(Hint: use generating function).

3) a) Find a closed form formula of  $p(n | \text{ at most 2 parts})$ .

b) Find a closed form formula of  $p(n | \text{ parts in } \{1, 2\})$ .

4) Find the set  $A$  such that

$p(n | \text{ no part appears more than twice}) = p(n | \text{ parts in } A)$   
for all  $n \geq 1$ .

5) In the 27-letter alphabet (with blank = 26), use the affine enciphering transformation with key  $a = 13$ ,  $b = 9$  to encipher the message "HELP ME".