SOLUTION 3

1. Solution

Problem 1 To show there are infinitely many primes of the form $6n+5$.

Proof:

Note: $(6n+1)(6m+1) = 36mn + 6n + 6m + 1 = 6(6mn + n + m) + 1$.
$(6n+3)(6m+3) = 36mn + 6n + 6m + 9 = 6(6mn + n + m + 1) + 3$.
$(6n+1)(6m+3) = 36mn + 6n + 6m + 3 = 6(6mn + n + m) + 3$.

From the calculation above, the product of numbers in the form $6n+1$ and $6n+3$ will also be in the form $6n+1$ or $6n+3$.

We prove by contradiction. Assume there are only finite prime of the form $6n+5$.

Let denote these prime by p_i. We have that $p_1 = 5, p_2 = 11, p_3 = 23, ..., p_k$.

Consider $Q := 6p_2p_3...p_k + 5$.

Case 1: Q is a prime. We have a new prime of the form $6n+5$. Contradiction.

Case 2: Q is composite. Then the prime factors of Q are odd numbers and must be in form $6n+1, 6n+3$ or $6n+5$.

However at least one of the prime factor of Q must be in the form $6n+5$ since the product of the numbers of the form $6n+1$ and $6n+3$ must only be in the form $6n+1$ or $6n+3$.

On the other hand, the prime of the form $6n+5$ could not divide Q. Contradiction again for case 2. □

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Problem 2 Set $a := \lceil \sqrt{n} \rceil$. Then check whether $\sqrt{a^2 - n}$ is an integer.

If yes then we can factor n as $n = (a - b)(a + b)$ where $b = \sqrt{a^2 - n}$.

If no, increase a by 1 and repeat the step.

a) $143 = 11 \cdot 13$.
b) $46009 = 139 \cdot 331$.
c) $3200399 = 1601 \cdot 1999$.

Problem 3

My bad, the problem is incorrect. I already corrected it and put in HW4.

Problem 17 page 132

To show the final digit of $2^{2^n} + 1$ is 7 for $n \geq 2$.

First show that the final digit of 2^{2^n} is 6 for $n \geq 2$ by using induction.

Base case: $2^{2^2} = 2^4 = 16$ which indeed has the last digit 6.

Induction step: Assume 2^{2^k} has the last digit 6 for all $k < n$.

Now consider 2^{2^n},

$2^{2^n} = 2^{(2^{n-1})2}$. Since $2^{2^{n-1}}$ has the final digit 6, $2^{(2^{n-1})2}$ also has the final digit 6.

Hence $2^{2^n} + 1$ has the final digit 7. $\square$

Problem 20 page 132

Find all the prime of the form $2^{2^n} + 5$.

We want to show that $2^{2^n} + 5$ is prime when $n = 0$ and otherwise has 3 as a factor.

$2^{2^0} + 5 = 7$ which is prime. Now we consider case when $n > 1$.

We see that $2^2 \equiv 1 \bmod 3$.

Therefore $2^{2^n} \equiv 2^{(2)2^{n-1}} \equiv 1^{2^{n-1}} \equiv 1 \bmod 3$.

So $2^{2^n} + 5 \equiv 1 + 5 \equiv 0 \bmod 3$. $\square$