SOLUTION 8

1. Solution

Problem 1

Most of people got this problem right. We make a bijective according to the algorithm discussed during the class.

Problem 2

First, the generating function of
$p(n|\text{distinct parts and each part} \equiv \pm 1 \bmod 3)$.

$F(q) = (1+q)(1+q^2)(1+q^4)(1+q^5)... = \prod_{i=0}^{\infty}(1+q^{3i+1})(1+q^{3i+2})$.

Second, the generating function of
$p(n|\text{parts are} \equiv \pm 1 \bmod 6)$.

$G(q)$

$= (1+q+q^2+q^3...)(1+q^5+q^{10}+...)(1+q^7+q^{14}+...)(1+q^{11}+q^{22}+...)...$

$= \prod_{i=0}^{\infty} \frac{1}{(1-q^{6i+1})(1-q^{6i+5})}$.

To show: $F(q) = G(q)$

$F(q) = \prod_{i=0}^{\infty}(1+q^{3i+1})(1+q^{3i+2})$
$= \prod_{i=0}^{\infty}(1+q^{3i+1})(1+q^{3i+2}) \prod_{i=0}^{\infty} \frac{(1-q^{3i+1})(1-q^{3i+2})}{(1-q^{3i+1})(1-q^{3i+2})}$
$= \prod_{i=0}^{\infty}(1-q^{6i+2})(1-q^{6i+4}) \prod_{i=0}^{\infty} \frac{1}{(1-q^{3i+1})(1-q^{3i+2})}$
$= \prod_{i=0}^{\infty}(1-q^{6i+2})(1-q^{6i+4}) \prod_{i=0}^{\infty} \frac{1}{(1-q^{6i+1})(1-q^{6i+4})(1-q^{6i+2})(1-q^{6i+5})}$
$= \prod_{i=0}^{\infty} \frac{1}{(1-q^{6i+1})(1-q^{6i+5})}$
$= G(q) \quad \square$.

Problem 3

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a) $p(n|\text{at most 2 parts}) = \lfloor \frac{n}{2} \rfloor + 1$.

b) $p(n|\text{ parts in } \{1,2\}) = \lfloor \frac{n}{2} \rfloor + 1$.

Problem 4

After making a table discussed in the class,
$A =$ set of all positive integers but multiples of 3.

Problem 5

Let $f(P) = 13P + 9 \bmod 27$.
Apply this function to the 1-letter block "HELP ME", we have the ciphertext "THRPXDH".