

NUMBER THEORY: CLASS 20

1. EXERCISE

- 1) Show that if n is an odd perfect number, then $n \neq p^a$, for some prime p and positive integer a .
- 2) Verify numerically for $n = 24$ that $C_d = \phi(n/d)$ for each divisor d of n . (C_d is as defined in class.)

For problems 3 and 4 below:

A positive integers n is called *superperfect* if $\sigma(\sigma(n)) = 2n$.

- 3) Show that if $n = 2^q$, where $2^{q+1} - 1$ is prime, then n is superperfect.
- 4) Show that every even perfect superperfect number is of the form $n = 2^q$, where $2^{q+1} - 1$ is prime.