

NUMBER THEORY: CLASS 3

1. EXERCISE

1) In this problem, we will learn to use maple program to solve Fibonacci identities.

i) Open the Classic Maple program.

ii) Command *rsolve* can be used to solve the recurrence relation. Try

`rsolve({g(n) = g(n - 1) + g(n - 2), g(1) = 1, g(2) = 1}, g);`

(You will obtain the formula of Fibonacci numbers).

iii) We now define the function of Fibonacci formula. Try

`f := n -> 1/sqrt(5) * (((1 + sqrt(5))/2)^n - ((1 - sqrt(5))/2)^n);`

iv) Try the command below to solve the Fibonacci identities $f_{n+3} + f_n = 2f_{n+2}$:

`simplify(f(n + 3) + f(n) - 2f(n + 2));`

v) Solve the following identities

1) $f_{2n} = f_n^2 + 2f_{n-1}f_n$.

2) $f_{n+1}f_{n-1} - f_n^2 = (-1)^n$.

2) Use Sieve method to find all the prime up to 70.

3) Use the Euclidean algorithm to find $\gcd(58, 24)$ and write the gcd as a linear combination of 58 and 24.