FORMULA SHEET

1. Chapter 2

Newton's Method:

$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}, \quad n \geq 1.$

Secant Method:

$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1}-p_{n-2})}{f(p_{n-1})-f(p_{n-2})}, \quad n \geq 2.$

Improved Newton's Method:

$p_n = p_{n-1} - \frac{f(p_{n-1})f'(p_{n-1})}{f'(p_{n-1})^2-f(p_{n-1})f''(p_{n-1})}, \quad n \geq 1.$

Aitken's Δ^2 Method:

$\hat{p_n} = p_{n-1} - \frac{(p_{n+1}-p_n)^2}{p_{n+2}-2p_{n+1}+p_n}$, given sequence p_n.

Steffensen's Method:

$p_0^{(n)} = p_0^{(n-1)} - \frac{(p_1^{(n-1)}-p_0^{(n-1)})^2}{p_2^{(n-1)}-2p_1^{(n-1)}+p_0^{(n-1)}}$, given $p_0 \quad (= p_0^{(0)})$.

where $p_1^{(n-1)} = g(p_0^{(n-1)})$ and $p_2^{(n-1)} = g(p_1^{(n-1)})$.

2. Chapter 3

Lagrange Interpolating polynomial:

Given $(x_0, y_0), (x_1, y_1), ..., (x_k, y_k)$.

$L_i(x) = \frac{(x-x_0)(x-x_1)...(x-x_{i-1})(x-x_{i+1})...(x-x_k)}{(x_i-x_0)(x_i-x_1)...(x_i-x_{i-1})(x_i-x_{i+1})...(x_i-x_k)}.$

$P(x) = y_0L_0(x) + y_1L_1(x) + ... + y_kL_k(x).$

Theorem 3.3: let $f \in C^{k+1}[a, b]$ and let $P(x)$ be the Lagrange Interpolating polynomial degree k in $[a, b]$. Then for each $x \in [a, b]$, there exists $\xi(x)$ in (a, b) such that

$f(x) = P(x) + \frac{f^{(k+1)}(\xi(x))}{(k+1)!}(x - x_0)(x - x_1)...(x - x_k).$

Neville's Method:

$P_{0,1,\ldots,k}(x) = \frac{(x-x_j)P_{0,1,\ldots,j-1,j+1,\ldots,k}(x)-(x-x_i)P_{0,1,\ldots,i-1,i+1,\ldots,k}(x)}{x_i-x_j}$
for any $i, j, i \neq j$.

Hermite Polynomial:

Given $(x_0, y_0), (x_1, y_1), ..., (x_k, y_k)$ and $(x_0, y'_0), (x_1, y'_1), ..., (x_k, y'_k)$.

$\widetilde{H}(x) = \sum_{i=0}^{k} y_i H_i(x) + \sum_{i=0}^{k} y'_i \hat{H}_i(x)$.

where $H_i(x) = [1 - 2(x - x_i)L'_i(x_i)]L_i^2(x)$,

$\hat{H}_i(x) = (x - x_i)L_i^2(x)$,

and $L_i(x) = \frac{(x-x_0)(x-x_1)...(x-x_{i-1})(x-x_{i+1})...(x-x_k)}{(x_i-x_0)(x_i-x_1)...(x_i-x_{i-1})(x_i-x_{i+1})...(x_i-x_k)}$.

Moreover, if $f \in C^{2n+2}[a + b]$, then

$f(x) = \widetilde{H}(x) + \frac{(x-x_0)^2...(x-x_n)^2}{(2n+2)!} f^{(2n+2)}(\xi)$ for some $\xi \in (a, b)$.

Cubic Spline Interpolation:

Definition:

Given a function f defined on $[a, b]$ with a set of points $a = x_0, ..., x_k = b$, a cubic spline interpolation S for f is a function that satisfies the following conditions:

a) $S(x)$ is a cubic polynomial, denoted $S_i(x)$, on the subinterval $[x_i, x_{i+1}]$ for each $i = 0, ..., k - 1$.

b) $S_i(x_i) = y_i$ and $S_i(x_{i+1}) = y_{i+1}$ for each $i = 0, ..., k - 1$.

c) $S'_i(x_{i+1}) = S'_{i+1}(x_{i+1})$ for each $i = 0, ..., k - 2$.

d) $S''_i(x_{i+1}) = S''_{i+1}(x_{i+1})$ for each $i = 0, ..., k - 2$.

e) One of the choice of boundary conditions is satisfied:

(i) $S''(x_0) = 0$ and $S''(x_k) = 0$ (free or natural boundary).

(ii) $S'(x_0) = f'(x_0)$ and $S'(x_k) = f'(x_k)$ (clamped boundary).