MATH 373: SOLUTION TO PRACTICE PROBLEM CLASS 20

1. Solution

1) Let $f(t,y) = y'(t) = 1 + (t-y)^2$.

We see that $f'(t,y) = \frac{\delta f(t,y)}{\delta t}$

$= 2(t-y)(1-y')$, (By chain rule, differentiate respect to t).

$= 2(t-y)(1-1-(t+y)^2)$.

$= -2(t-y)^3$.

So the approximation term is Taylor polynomial degree 2, we have,

Approximation term $= y + \frac{hy'(t)}{1!} + \frac{h^2 y''(t)}{2!}$.

$= y + \frac{hf(t,y)}{1!} + \frac{h^2 f'(t,y)}{2!}$.

$= y + h(1+(t-y)^2) + \frac{h^2(-2(t-y)^3)}{2}$.

So we get the recurrence of the approximations:

$w_i := w_{i-1} + h(1+(t_{i-1}-w_{i-1})^2) + \frac{h^2(-2(t_{i-1}-w_{i-1})^3)}{2}$.

In this problem $t_0 = 2, \quad t_1 = 2.5, \quad h = 0.5, \quad w_0 = y_0 = 1$.

You plug everything in you get.

$y(\frac{5}{2}) \approx w_1 = 1.75$.

$y(3) \approx w_2 = 2.42578125$.

Note: the exact solution of this problem is $y(t) = t + \frac{1}{1-t}$.

So $y(\frac{5}{2}) = \frac{5}{2} + \frac{1}{(1-\frac{5}{2})} \approx 1.83333$.

and $y(3) = 3 + \frac{1}{1-3} = 2.5$.

Date: August 9, 2007.