# Roles of Computer in Mathematics

Thotsaporn "Aek" Thanatipanonda

Mahidol University, International College

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# Introduction

The idea of having computer to help doing mathematics has been floating around for a long time.

Some famous works from the past related to computer:

1. Four color-theorem (Kenneth Appel and Wolfgang Haken, 1976)
2. Classification finite simple group (completed in 2004, computer-checked version was conducted in 2012)
3. Kepler conjecture: Sphere packing (long proof, need computer to check complicated inequalities).

**Past Goals:** Check parts of the proof by going through cases or solving inequalities.
This style is called "Computer assisted proof".

# Automated Proof

It has always been a dream of mathematician to have computer produced a theorem *all by itself*. Famous people brought it up from time to time. However this is very broad and I am not sure what it exactly means.

On another aspect, computer can already prove a rigorous theorem, thanks to the power of symbolic computation!

**New Goal:** Aim for **an automated proof technique** from the ground up. I will demonstrate my point by showing you two examples.

# Method of Guess and Check

**First Example: Average length of random walk**

**Question:**
Imagine yourself go to a casino with \$M in your pocket. You play a game where you have $1/2$ chance to win \$1 and $1/2$ chance to lose \$1. Find the average number of plays before your reach \$N or you go broke.
($N \geq M$, of course).

# Average length of random walk

**Solution:**
Let $W(M,N)$ be average number of random walk from M to 0 or N whichever first.

**Step1:** Set up the system of relations

$$
\begin{aligned}
W(0,N) &= 0, \\
W(M,N) &= 1 + \frac{1}{2}\cdot W(M-1,N) + \frac{1}{2}\cdot W(M+1,N), \\
&\qquad\qquad\qquad\qquad \text{for } 1 \le M \le N-1, \\
W(N,N) &= 0.
\end{aligned}
$$

# Average length of random walk

**Step2:** Guess the answer.

**Step3:** Check! Verify your answer with system of relations.

Done!

# WZ pairs to evaluate Binomial Identities

The second example is an excellent example of automated proving machine where computer do all the calculation and you can verify the proof by hand.

**Second Example: Binomial Identities**

**Theorem**

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1} \qquad , n \geq 0.$$

# Binomial Identities: $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$

**Solution:**
By dividing $n2^{n-1}$, we want to show

$$\sum_{k=0}^{n} \frac{k\binom{n}{k}}{n2^{n-1}} = 1.$$

**Step1:**
Let $F(n,k) = \dfrac{k\binom{n}{k}}{n2^{n-1}}$.
Let $G(n,k) = R(n,k) \cdot F(n,k)$ where $R(n,k)$ is the key to unlock this identity which is given by computer program.
This case $R(n,k) = \dfrac{k-1}{2(k-n-1)}$.

# Binomial Identities: $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$

**Step2:**

All you need to do is to verify the followings:

1. $F(n+1,k) - F(n,k) = G(n,k+1) - G(n,k)$
2. $\lim_{k\to\pm\infty} G(n,k) = 0.$

After summing both sides with $k$ from $-\infty$ to $\infty$, we have that

$$\sum_k F(n+1,k) = \sum_k F(n,k).$$

Conclude that $\sum_k F(n,k)$ is a constant in $n$.

# Binomial Identities: $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$

**Step3:**

Verify $\sum_k F(1,k) = \sum_{k=0}^{1} \frac{0\cdot\binom{1}{0} + 1\cdot\binom{1}{1}}{1\cdot 2^0} = 1.$

Done!

# Conclusion

Computer produced a theorem is possible. It already happened in some areas. But we must try to learn and make more progress bit by bit. We should aim for building up an algorithm to serve computer.

"Ask not what your *computer* can do for you, ask what you can do for your *computer*."