

Elementary Statistics: Solution to Homework 6

Solution

Page 379 Problem 6.20:

a)

Given $\mu = 1800$ and $\sigma = 200$.

b) $z = \frac{1500-1800}{200} = -1.5$. We have the proportion of account below \$1500 is 0.0668.

c) We look for the z value at the 90 percentile. We have $z = 1.282$. This gives $X = 200(1.282) + 1800 = 2056.4$.

Page 380 Problem 6.24:

Given $\mu = 464$ and $\sigma = 8$.

a) We first convert $X = 454$ and $X = 474$ to z -value.

$$X = 454 : z = \frac{454-464}{8} = -\frac{10}{8} = -1.25$$

$$X = 474 : z = \frac{474-464}{8} = \frac{10}{8} = 1.25.$$

We look at the table and find out that the area between $z = 1.25$ and $z = -1.25$ is $0.8944-0.1056 = 0.7888$.

b) Given the position at top 10 % , we need to find X .

Look at the table page 408, we find at 90 percentile, $z = 1.282$.
Now we solve the equation

$$1.282 = \frac{X-464}{8}.$$

We have $X = 1.282(8) + 464 = 474.256$.

c) If the standard deviation were halved, the data will accumulate more in the middle. As a result the proportion of cans between 454 and 474 grams will increase.

Page 382 Problem 6.32:

- a) We know the area under the density function adds up to 1. Therefore $1 = \frac{1}{2}6h$. Solving for the height h . $h = \frac{1}{3}$.
- b) Symmetric.
- c) The percentage $= \frac{1}{2}(3)\frac{1}{6} = \frac{1}{4} = 25\%$
- d) The significance level is α is the chance of making a Type I error.
We calculate the area of rejection region in H_0 . $A = \frac{1}{2}1\frac{1}{18} = \frac{1}{36}$.
- e) $\beta =$ area under the curve from 1 to 6 in $H_1 = 0.1+0.05+0.05+0.1+0.35 = 0.65$.
- f) According to the decision rule in part d), we accept H_0 .

Page 387 Problem 6.38:

a)

The height of rectangle is $\frac{1}{51-49} = \frac{1}{2}$.

- b) $A = \frac{1}{2}(51 - 49) = \frac{1}{2}$.
- c) Median = 50. $Q_1 = 49.5$ and $Q_3 = 50.5$.
Therefore $IQR = Q_3 - Q_1 = 50.5 - 49.5 = 1$.

Page 394 Problem 6.48:

a) $P(X = 4) = 0.05$.

b) $P(X \geq 1) = 0.95$.

Page 395 Problem 6.50:

- a) The proportion of families have one child is $1-(0.4+0.2+0.1+0.05)=0.25$.
- b) The proportion of families have at least one child is $0.25+0.2+0.1+0.05 = 0.60$.
- c) Skew to right. Therefore the answer is v) none of the above.

Page 398 Problem 6.60:

Most people got this problem wrong. The answer is c).

Part a) is wrong. Although the new set of scores has mean 0 but the new variance is less than 1. For the variance to be 1, you need to divide the difference by 10 instead of 100.

Part b) is also wrong. Since the problem did not mention the score has a normal distribution to start with.

Page 398 Problem 6.70:

a)

b) i) mean of $X = 0$.

ii) mean of $Y = -2$.

iii) Q_1 for $X = -1$.

iv) Q_3 for $X = 1$.

v) Q_1 for $Y = (-0.674)(2)-2 = -3.348$.

vi) Q_3 for $Y = (0.674)(2)-2 = -0.652$.

vii) Proportion ($X > 0$) = $\frac{1}{2}$.

viii) Proportion ($Y > 0$), we have $Z = \frac{0-(-2)}{2} = 1$. From the table page 406-407, the area = $1-0.8413 = 0.1587$.

Page 401 Problem 6.80:

I would agree if the original data has a normal distribution.

Page 428 Problem 7.15:

a) No, since there is a card *king of heart*.

b) Yes, since the card could not have heart and spade at the same time.

c) No, since there is a card *king of spade*.

Page 440 Problem 7.18:

a)

b) $P(A \cap B) = 0.1$

c) $P((A \cup B)^C) = 0.4.$