Elementary Statistics: Solution to Homework 7

Solution

Page 526 Problem 8.2:

a) Estimator I and II are unbiased. The true parameter is at the center of the distribution.
b) I would select Estimator I since it has less variability (only one peak).
c) Estimator III. Data accumulate most around the center.
d) Estimator I since it is unbiased and has less variability than Estimator II.

Page 526 Problem 8.3:

The answer is b).

Page 526 Problem 8.4:

a) i) decreases.
b) iii) stay the same.
c) iv) looks more and more like a normal distribution and becomes more and more tightly clustered about its mean.

Page 526 Problem 8.8:

Given p (the true proportion) $= 0.6$ and n (sample size) $= 40$.

The sampling distribution of the sampling proportions follows a normal distribution (after checking the conditions $np \geq 5$ and $n(1 - p) \geq 5$) with

$\mu_{\hat{p}} = 0.6$ and

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}} = \sqrt{\frac{0.6 \cdot 0.4}{40}} = 0.07746.$$

a) $\hat{p} = 0.75$ implies $Z = \dfrac{0.75 - 0.6}{0.07746} = 1.936.$
$P(\hat{p} \geq 0.75) = P(Z \geq 1.936) = 1 - 0.9738 = 0.0262.$

b) $\hat{p} = 0.25$ implies $Z = \dfrac{0.25 - 0.6}{0.07746} = -4.52.$
$P(\hat{p} \leq 0.25) = P(Z \leq -4.52) < 0.00001.$

From the above, it is quite unusual to find $\hat{p} < 0.25$.

Page 526 Problem 8.12:

a)

b) The sampling distribution of the sampling proportions follows a normal distribution with

$\mu_{\hat{p}} = 0.6$ and
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6 \cdot 0.4}{100}} = 0.049.$$

(i) There is a 68% chance that the sample proportion is between 0.551 and 0.649. (one standard deviation from the mean)
(ii) There is a 95% chance that the sample proportion is between 0.502 and 0.698. (two standard deviation from the mean)
(iii) It is almost certain that the sample proportion is between 0.453 and 0.747. (three standard deviation from the mean)

c) It is unlikely to get exactly a sample proportion of 0.50 since it is just one particular value.

d) i) The distribution would be thinner and taller with the center at the same mean.
ii) The range in part b) will decrease.