April 16, 2009

Elementary Statistics: Solution to Homework 8

Solution

Page 551 Problem 8.17:

False, the central limit theorem only valid when the sample is ≥ 30 or the original data is normally distributed.

Page 551 Problem 8.18:

a) Normal Distribution with $\mu_{\bar{x}} = 69$ and $\sigma_{\bar{x}} = \frac{2.5}{\sqrt{9}} = 0.9129$.
b) Normal Distribution with $\mu_{\bar{x}} = 69$ and $\sigma_{\bar{x}} = \frac{2.5}{\sqrt{100}} = 0.5$.
c) The distribution in part b) has smaller standard deviation hence is more accumulated around the center.

Page 553 Problem 8.25:

a) Histogram C

b) Histogram F

Page 553 Problem 8.26:

a) No, the median is not equal to 4. The area under the curve to the left of 4 is less than 0.5.
b) No, because the distribution is skewed to the left.
c) Histogram C because of the central limit theorem.

Page 559 Problem 8.38:

Histogram A by the Central Limit Theorem: the distribution of $\bar{X}$ is normally distributed with the center at \$60,000 .

Page 559 Problem 8.40:

a) By the empirical rule $2\sigma = 20$. This implies $2\frac{100}{\sqrt{n}} = 20$. Hence $n = 100$.
b) Similarly $n = 400$.
c) $n = 1600$.
d) The bigger the sample size, the better accuracy at estimating a population mean.

Page 580 Problem 9.3:

d) is equal to α, by definition.

Page 580 Problem 9.4:

d) Type II error.

Page 580 Problem 9.5:

To reject H_0, the p-value need to be less than α.
Hence the answer is b) p-value $= 0.005$.

Page 580 Problem 9.6:

a) This is a two-sided test since H_1 say the percentage is not equal to 70%.

b) The point estimate is the sample proportion $\hat{p}$. $\hat{p} = \dfrac{64}{106} = 0.6038$.

c) $\sigma = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.7 \cdot (1-0.7)}{106}} = 0.0445$.
$Z = \dfrac{\hat{p} - p}{\sigma} = \dfrac{0.6038 - 0.7}{0.0445} = -2.161$.

We use the table to find the p-value.
The p-value $= 2 \cdot 0.0154 = 0.0308$, since the test is two-sided.

d) Since the p-value is less than α, we reject the null hypothesis.

Page 581 Problem 9.8:

Let p be the unemployment rate for the city.

a) $H_0 : p = 0.05$
$H_1 : p > 0.05$.
Note: In this chapter H_0 always proportion p equals some numbers.

b) $\hat{p} = \dfrac{125}{2000} = 0.0625$.
$\sigma = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.05 \cdot (1-0.05)}{2000}} = 0.00487$.
The test statistic is $Z = \dfrac{\hat{p} - p}{\sigma} = \dfrac{0.0625 - 0.05}{0.00487} = 2.5649$.

We use the table to find the p-value.

The p-value $= 1 - 0.9949 = 0.0051$. We use $1 - A$ because the rejection region is one-sided to the right.

c) Since the p-value is less than α.
We reject H_0 and conclude that the unemployment rate in this city is higher than 0.05.

Page 582 Problem 9.12:

Let p be the proportion of all the workers who willing to work fewer hours to obtain more free time.

$p = 0.2$, $\hat{p} = 0.14$ and $n = 600$.

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.2 \cdot (1 - 0.2)}{600}} = 0.0163.$$

The test statistic is $Z = \dfrac{\hat{p} - p}{\sigma} = \dfrac{0.14 - 0.2}{0.0163} = -3.674.$

There is no value $Z = -3.674$. But we know the p-value is very small.
Again since the p-value is less than α. We reject H_0 and conclude that p is less than 0.2.