April 19, 2009

Elementary Statistics: Solution to Homework 9

Solution

Page 595 Problem 9.20:

$\hat{p} = 0.36$ and $n = 400$.

a) The confidence interval I is $(\hat{p} - z^*\sqrt{\dfrac{\hat{p}\cdot(1-\hat{p})}{n}}, \hat{p} + z^*\sqrt{\dfrac{\hat{p}\cdot(1-\hat{p})}{n}})$
$= (0.36 - 2\sqrt{\dfrac{(0.36)(0.64)}{400}}, 0.36 + 2\sqrt{\dfrac{(0.36)(0.64)}{400}})$
=(0.312,0.408).

b) We are 95% *confident* that this interval I contains the population proportion p.
If we repeatedly took the samples and computer 95% confidence intervals, we would expect approximately 95% of these intervals contain the population proportion p.

c) The interval is still the same since we do not use the population size in our calculation.

d) The bigger the sample size, the smaller confidence interval.

Page 595 Problem 9.22:

$\hat{p} = \frac{20}{280} = 0.0714$ and $n = 280$.

a) $I_1 = (0.0714 - 1.645\sqrt{\frac{(0.0714)(1-0.0714)}{280}}, 0.0714 + 1.645\sqrt{\frac{(0.0714)(1-0.0714)}{280}})$
=(0.0461,0.0967).

b) $I_2 = (0.0714 - 2\sqrt{\frac{(0.0714)(1-0.0714)}{280}}, 0.0714 + 2\sqrt{\frac{(0.0714)(1-0.0714)}{280}}) = (0.0406, 0.1022)$.

c) I_2 is wider.

Page 597 Problem 9.28:

The answer is d).

Page 597 Problem 9.29:

a) The distribution of $\hat{p}$ is approximately $N(p, \sqrt{\dfrac{p(1-p)}{n}})$.
b) No, the bias comes from the selection method not the sample size.
c) Yes, $\sigma_{\hat{p}}$ gets smaller as n increases.
d) We are 95% *confident* that this interval (0.51,0.55) contains the proportion of people who vote for the Democratic candidate p.

Page 633 Problem 10.1:

a) H_0 : The mean temperature in Wayne County for the month of January is equal to 33 degrees.
H_1 : The mean temperature in Wayne County for the month of January is below 33 degrees.

b) H_0 : The mean age of medical students at WSU is equal to 26 years.
H_1 : The mean age of medical students at WSU is more than 26 years.

c) H_0 : The mean score on an entrance exam is the same as the target score of 200 set by the exam developers.
H_1 : The mean score on an entrance exam differs from the target score of 200 set by the exam developers.

Page 633 Problem 10.4:

H_0: The IQ score is 100.
H_1: The IQ score is higher than 100.

Given $\bar{x} = 114, n = 9$ and $\sigma = 15$.

Since the original population is normally distributed, we can assume $\bar{x}$ is normally distributed.

The test statistic:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{114 - 100}{\frac{15}{\sqrt{9}}} = \frac{14}{5} = 2.8.$$

From the table page 407, the p-value is 1-0.9974 = 0.0026.

Since the p-value is less than $\alpha = 0.01$, we reject the null hypothesis.
We conclude that the IQ score is higher than 100.

Page 634 Problem 10.8:

a) The p-value is about 0.025.

b) The p-value is about 0.13.

c) The p-value is about $2 \cdot 0.008 = 0.016$.

Page 634 Problem 10.10:

H_0: The average speed on a particular highway is 70 mph.
H_0: The average speed on a particular highway exceeds 70 mph.

Given $\bar{x} = 73.2, \quad n = 16$ and the sample standard deviation $s = 5.1$.

More data (the sample size at least 30) might give a more accurate picture of this research.

The test statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{73.2 - 70}{\frac{5.1}{\sqrt{16}}} = 2.5098.$$

From the table page 668 with degree of freedom = 15, the p-value is about 0.015.

Since the p-value is less than $\alpha = 0.05$, we reject the null hypothesis.
We conclude that the average speed on this particular highway exceeds 70 mph.